Problem sheet for the lecture

MA2008 – Linear Algebra II

Sheet 5 Autumn Semester 2019

Some Matrix Algebra

Exercise 1 (Vectors and matrices).
Compute each of the following matrix-vector products (the last one has entries in $\mathbb{C}$, the first two in $\mathbb{R}$):

$$\begin{pmatrix} 1 & 2 \\ -3 & -4 \\ 5 & -6 \end{pmatrix} \begin{pmatrix} -2 \\ 7 \end{pmatrix} \quad , \quad \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -3 \\ 4 \end{pmatrix} \quad , \quad \begin{pmatrix} 1 & 1+i & 4 \\ -i & -2 & 3-2i \end{pmatrix} \begin{pmatrix} 2+i \\ -i \\ 1 \end{pmatrix}$$

Exercise 2 (Matrix products).
Let $A, B \in M_{3\times 3}(\mathbb{R})$, $C \in M_{4\times 3}(\mathbb{R})$ be the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \quad , \quad B = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad C = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Compute the following matrix products:

$$A \cdot B \quad , \quad B \cdot A \quad , \quad C \cdot A \quad , \quad A \cdot C^t \quad , \quad C \cdot A^t \ .$$

Exercise 3 (Eigenvalues and eigenvectors).
Find all eigenvalues and eigenvectors of each of the following matrices over $\mathbb{C}$:

$$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \quad , \quad \begin{pmatrix} i & 2+i \\ 0 & 1 \end{pmatrix} \quad , \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Exercise 4 (Eigenvectors and geometry).
Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T(x, y) = (y, x)$$

Let $\alpha = \{e_1, e_2\}$ be the standard basis.

a) Write down the matrix representation $A = [T]_\alpha^\alpha$ for T with respect to α.

b) Show that the eigenvalues of T are $\lambda_1 = 1$ and $\lambda_2 = -1$ and compute corresponding eigenvectors.

c) Find a geometric interpretation for the eigenvectors of T.