

The strictification theorem

$(\mathcal{C}, \otimes, a, \lambda, \rho)$ monoidal category

construct \mathcal{F} with objects: lists of objects in \mathcal{C} of finite length.

have a map $f: ob(\mathcal{F}) \rightarrow ob(\mathcal{C})$

$$\begin{aligned} [V_1, V_2, \dots, V_n] &\mapsto V_1 \otimes (V_2 \otimes (V_3 \otimes \dots \otimes V_n) \dots) \\ [] &\mapsto 1 \end{aligned}$$

morphisms of \mathcal{F}

$$hom_{\mathcal{F}}(A, B) = hom_{\mathcal{C}}(f(A), f(B))$$

monoidal structure on \mathcal{F} : concatenation of objects (denoted \cdot)

note that there is a unique isomorphism rebracketing $f(A \cdot B)$ to $f(A) \otimes f(B)$ due to the coherence thm.

so given $g: A \rightarrow C$, $h: B \rightarrow D$ we set

$$g \otimes h : f(A \cdot B) \xrightarrow{\uparrow \text{rebracketing}} f(A) \otimes f(B) \longrightarrow f(C) \otimes f(D) \xrightarrow{\uparrow \text{rebracketing}} f(C \cdot D)$$

\hookrightarrow mono \rightarrow strict monoidal category \mathcal{F} .

functor $F: \mathcal{F} \rightarrow \mathcal{C}$ is given by $F(A) = f(A)$ identity on morphisms

$$F_{(A,B)} : F(A \cdot B) \longrightarrow F(A) \otimes F(B) \text{ is rebracketing}$$

$$F_{\cdot} = id \text{ on } F(1) = 1$$